

# Thinned Concentric Circular Array Antennas Synthesis using Improved Particle Swarm Optimization

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**Abstract**— Circular antenna array has gained immense popularity in the field of communications nowadays. It has proved to be a better alternative over other types of antenna array configuration due to its all-azimuth scan capability, and a beam pattern which can be kept invariant. This paper is basically concerned with the thinning of a large multiple concentric circular ring arrays of uniformly excited isotropic antennas based on Improved Particle Swarm Optimization (IPSO) method. In this paper a 9 ringed Concentric Circular Antenna Array (CCAA) with central element feeding is considered. The computational results show that the number of antenna array elements can be brought down from 279 to 147 with simultaneous reduction in Side Lobe Level of about 20 dB with a fixed half power beamwidth.

**Keywords**- Concentric Circular Antenna Arrays; Particle Swarm Optimization; Thinning; Sidelobe Level

## I. INTRODUCTION

Concentric Circular Antenna Array (CCAA) has several interesting features that make it indispensable in mobile and communication applications. CCAA [1-11] has received considerable interest for its symmetry and compactness in structure. A concentric circular array antenna is an array that consists of many concentric rings of different radii and a number of elements on its circumference. Since a concentric circular array does not have edge elements, directional patterns synthesized with a concentric circular array can be electronically rotated in the plane of the array without a significant change of the beam shape. CCAA provides great flexibility in array pattern synthesis and design both in narrow band and broadband applications. It is also favoured in direction of arrival (DOA) applications since it provides almost invariant azimuth angle coverage. Uniform CCA is the CCA where all the elements in the array are uniformly excited and the inter-element spacing in individual ring is kept almost half of the wavelength. For larger number of rings with uniform excitations, the side lobe in the UCCA drops to about 17.5 dB. Lot of research has gone into optimizing antenna structures such that the radiation pattern has low sidelobe level. This very fact has driven researchers to optimize the CCAA design.

Although uniformly excited and equally spaced antenna arrays have high directivity at the same time they suffer from high side lobe level. Reduction in side-lobe level can be brought about in either of the following ways, either by keeping excitation amplitudes uniform but changing the position of antenna elements or by using equally spaced array with radially tapered amplitude distribution. These processes are referred to as thinning. Thinning not only reduces side lobe level but also brings down the cost of manufacturing by decreasing the number of antenna elements [12, 13]. There are various global optimization tools for thinning such as Genetic Algorithms (GA) [8], Particle Swarm Optimization (PSO) [14-17] etc. The PSO algorithm has proved to be a better alternative to other evolutionary algorithms such as Genetic Algorithms (GA), Ant Colony Optimization (ACO) etc. in handling certain kinds of optimization problems. This paper proposes a method for thinning a large multiple concentric circular ring arrays of isotropic antennas based on PSO. The rest of the paper is organized as follows: In section II, the general design equations for the CCAA are stated. Then, in section III, a brief introduction for the PSO is presented. Computational results are presented in section IV. Finally the paper concludes with a summary of the work in section V.

## II. DESIGN EQUATION

Fig. 1 shows the general configuration of CCAA with  $M$  concentric circular rings, where the  $m^{\text{th}}$  ( $m = 1, 2, \dots, M$ ) ring has a radius  $r_m$  and the corresponding number of elements is  $N_m$ . If all the elements (in all the rings) are assumed to be isotropic sources, the radiation pattern of this array can be written in terms of its array factor only. Referring to Fig. 1, the far field pattern of a thinned CCAA in  $x$ - $y$  plane may be written as [17]:

$$AF(\theta, \phi) = \sum_{m=1}^M \sum_{i=1}^{N_m} I_{mi} \exp[j(kr_m \sin \theta \cos(\phi - \phi_{mi}) + \alpha_{mi})] \quad (1)$$

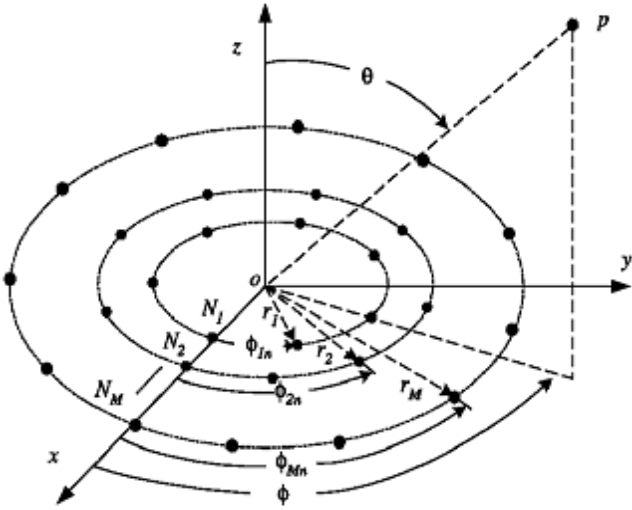


Figure 1. Concentric circular antenna array (CCAA).

where  $I_{mi}$  denotes current excitation of the  $i^{\text{th}}$  element of the  $m^{\text{th}}$  ring,  $k = 2\pi / \lambda$ ;  $\lambda$  being the signal wave-length. If the elevation angle,  $\phi = \text{constant}$ , then (1) may be written as a periodic function of  $\theta$  with a period of  $2\pi$  radian i.e. the radiation pattern will be a broadside array pattern. The azimuth angle to the  $i^{\text{th}}$  element of the  $m^{\text{th}}$  ring is  $\phi_{mi}$ . The elements in each ring are assumed to be uniformly distributed.  $\phi_{mi}$  and  $\alpha_{mi}$  are also obtained from [13] as:

$$\phi_{mi} = 2\pi((i-1)/N_m) \quad (2)$$

$$\alpha_{mi} = -Kr_m \sin \theta_0 \cos(\phi - \phi_{mi}) \quad (3)$$

$\theta_0$  is the value of  $\theta$  where peak of the main lobe is obtained. After defining the array factor, the next step in the design process is to formulate the objective function which is to be minimized. The objective function “Cost Function” ( $CF$ ) may be written as (4):

$$CF = W_{F1} \times \frac{|AF(\theta_{msl1}, I_{mi}) + AF(\theta_{msl2}, I_{mi})|}{|AF(\theta_0, I_{mi})|} + W_{F2} \times (FNBW_{computed} - FNBW(I_{mi}=1)) \quad (4)$$

$FNBW$  is an abbreviated form of first null beamwidth, or, in simple terms, angular width between the first nulls on either side of the main beam.  $CF$  is computed only if  $FNBW_{computed} < FNBW(I_{mi}=1)$  and corresponding solution of current excitation weights is retained in the active population otherwise not retained.  $W_{F1}$  (unitless) and  $W_{F2}$  ( $\text{radian}^{-1}$ ) are the weighting factors.  $\theta_0$  is the angle where the highest maximum of central lobe is attained in  $\theta \in [-\pi, \pi]$ .  $\theta_{msl1}$  is the angle where the maximum sidelobe ( $AF(\theta_{msl1}, I_{mi})$ ) is attained in the lower band and  $\theta_{msl2}$  is the

angle where the maximum sidelobe ( $AF(\theta_{msl2}, I_{mi})$ ) is attained in the upper band.  $W_{F1}$  and  $W_{F2}$  are so chosen that optimization of SLL remains more dominant than optimization of  $FNBW_{computed}$  and  $CF$  never becomes negative. In (4) the two beamwidths,  $FNBW_{computed}$  and  $FNBW(I_{mi}=1)$  basically refer to the computed first null beamwidths in radian for the non-uniform excitation case and for uniform excitation case respectively. Minimization of  $CF$  means maximum reductions of SLL both in lower and upper sidebands and lesser  $FNBW_{computed}$  as compared to  $FNBW(I_{mi}=1)$ . The evolutionary optimization techniques employed for optimizing the current excitation weights resulting in the minimization of  $CF$  and hence reductions in both SLL and  $FNBW$  are described in the next section. In this case,  $I_{mi}$  is 1 if the  $mi$ -th element is turned “on” and 0 if it is “off.” All the elements have same excitation phase of zero degree.

An array taper efficiency can be calculated from

$$\eta_{ar} = \frac{\text{number of elements in the array turned on}}{\text{total number of elements in the array}}$$

### III. EVOLUTIONARY TECHNIQUES EMPLOYED

#### A. Particle Swarm Optimization (PSO)

PSO is a flexible, robust population-based stochastic search/optimization technique with implicit parallelism, which can easily handle with non-differential objective functions, unlike traditional optimization methods. PSO is less susceptible to getting trapped on local optima unlike GA, Simulated Annealing, etc. Eberhart and Shi [14] developed PSO concept similar to the behavior of a swarm of birds. PSO is developed through simulation of bird flocking in multidimensional space. Bird flocking optimizes a certain objective function. Each particle knows its best value so far (pbest). This information corresponds to personal experiences of each particle. Moreover, each particle knows the best value so far in the group (gbest) among pbests. Namely, each particle tries to modify its position using the following information:

- The distance between the current position and pbest.
- The distance between the current position and gbest.

Mathematically, velocities of the particles are modified according to the following equation [15, 16]:

$$V_i^{(k+1)} = w * V_i^k + C_1 * rand_1 * (pbest_i - S_i^k) + C_2 * rand_2 * (gbest - S_i^k) \quad (5)$$

where  $V_i^k$  is the velocity of  $i^{\text{th}}$  particle at  $k^{\text{th}}$  iteration;  $w$  is the weighting function;  $C_j$  is the weighting factor;  $rand_i$  is the random number between 0 and 1;  $S_i^k$  is the current position of particle  $i$  at iteration  $k$ ;  $pbest_i$  is the personal best of particle  $i$ ;  $gbest$  is the group best among all pbests for the group. The searching point in the solution space can be modified by the following equation:

$$S_i^{(k+1)} = S_i^k + V_i^{(k+1)} \quad (6)$$

The first term of (5) is the previous velocity of the particle. The second and third terms are used to change the velocity of the particle. Without the second and third terms, the particle will keep on “flying” in the same direction until it hits the boundary. Namely, it corresponds to a kind of inertia and tries to explore new areas. The values of  $w$ ,  $C_1$  and  $C_2$  are given in the next section.

### B. Improved Particle Swarm Optimization (IPSO)

The global search ability of traditional PSO is very much enhanced with the help of the following modifications. This modified PSO is termed as IPSO [15, 16]. i) The two random parameters  $rand_1$  and  $rand_2$  of (5) are independent. If both are large, both the personal and social experiences are over used and the particle is driven too far away from the local optimum. If both are small, both the personal and social experiences are not used fully and the convergence speed of the technique is reduced. So, instead of taking independent  $rand_1$  and  $rand_2$ , one single random number  $r_1$  is chosen so that when  $r_1$  is large,  $rand_1$  is small and vice versa. Moreover, to control the balance of global and local searches, another random parameter  $r_2$  is introduced. For birds flocking for food, there could be some rare cases that after the position of the particle is changed according to (6), a bird may not, due to inertia, fly toward a region at which it thinks is the most promising for food. Instead, it may be leading toward a region which is in the opposite direction of what it should fly in order to reach the expected promising regions. So, in the step that follows, the direction of the bird's velocity should be reversed in order for it to fly back into the promising region. is introduced for this purpose. Both cognitive and social parts are modified accordingly. Finally, the modified velocity of  $j^{th}$  component of  $i^{th}$  particle is expressed as follows:

$$V_i^{(k+1)} = r_2 * sign(r_3) * V_i^k + (1 - r_2) * C_1 * r_1 * \{pbest_i^k - S_i^k\} + (1 - r_2) * C_2 * (1 - r_1) * \{gbest^k - S_i^k\} \quad (7)$$

where  $r_1$ ,  $r_2$  and  $r_3$  are the random numbers between 0 and 1;

$S_i^k$  is the current position of particle  $i$  at iteration  $k$ ;  $pbest_i^k$  is the personal best of  $i^{th}$  particle at  $k^{th}$  iteration;  $gbest^k$  is the group best among all  $pbests$  for the group at  $k^{th}$  iteration. The searching point in the solution space can be modified by the following equation (6).

$sign(r_3)$  is a function defined as:

$$sign(r_3) = -1 \text{ when } r_3 < 0.05, \\ = 1 \text{ when } r_3 > 0.05. \quad (8)$$

## IV. COMPUTATIONAL RESULTS

This section gives the computational results for the CCAA synthesis obtained by IPSO technique. Each CCAA maintains a fixed optimal inter-element spacing between the elements in each

ring. The limits of the radius of a particular ring of CCAA are decided by the product of number of elements in the ring and the inequality constraint for the inter-element spacing,  $d$ , ( $d \in [\lambda/2, \lambda]$ ). For all the cases,  $\theta_0 = 0^\circ$  is considered so that the peak of the main lobe starts from the origin. Since PSO techniques are sometimes quite sensitive to certain parameters, the simulation parameters should be carefully chosen. Best chosen maximum population pool size= 120, maximum iteration cycles for optimization= 50, and  $C_1 = C_2 = 1.5$ .

Fig. 2 is a diagram of a 279 element concentric ring array. Nine rings ( $N_1, N_2, \dots, N_9$ ) are considered for synthesis having (6, 12, 18, 25, 31, 37, 43, 50, 56) elements with central element feeding.

For this case  $r_m = m \times \frac{\lambda}{2}$  and inter-element spacing in each

rings are  $d_m \cong \frac{\lambda}{2}$ . The number of equally spaced elements in ring  $m$  is given by

$$N_m = \frac{2\pi r_m}{d_m} = 2\pi m \quad (9)$$

Since the number of elements must be an integer, the value in (9)

must be rounded up or down. To keep  $d_m \geq \frac{\lambda}{2}$ , the digits to

the right of the decimal point are dropped. Table I. lists the ring spacing and number of elements in each ring for a uniform concentric ring array with nine rings as shown in Figure 2.

The IPSO generates a set of optimal uniform current excitation weights for each synthesis set of CCAA.  $I_{mi}$  is 1 if the  $mi$ -th element is turned “on” and 0 if it is “off”. The optimal results are shown in Tables II- III.

### A. Analysis of Radiation Patterns of CCAA

Fig. 3 depicts the substantial reductions in SLL with optimal current excitation weights after thinning, as compared to the case of uniform current excitation weights and of fully populated array (considering fixed inter-element spacing,  $dH \sim \lambda/2$ ).

As seen from Table III and Fig. 3, the SLL reduces to  $H \sim 20$  dB for the given Set, with respect to -17.4 dB for uniform excitation and  $dH \sim \lambda/2$ . Further, the above improvement is achieved for an array of  $H \sim 53\%$  elements turned off.

The above results reveal that the thinned 9 rings CCAA set with central element feeding yield reductions in SLL as compared to the fully populated same array.

### B. Convergence Profile of IPSO

The minimum  $CF$  values are plotted against the number of iteration cycles to get the convergence profiles as shown in Fig. 4. The IPSO technique yield convergence to the minimum SLL in less than 25 iterations. All computations were done in MATLAB 7.5 on core (TM) 2 duo processor, 3.00 GHz with 2 GB RAM.

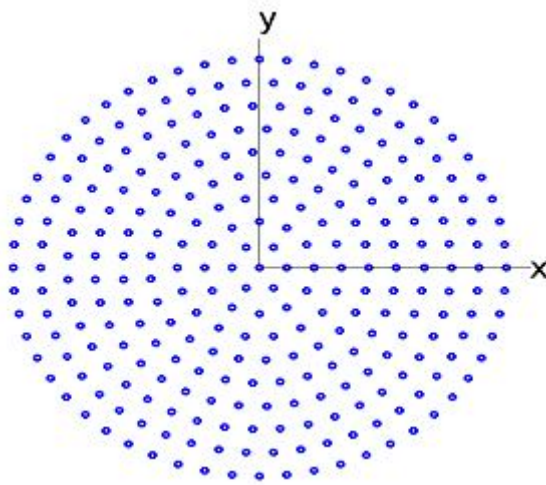


Figure 2. Concentric ring array with nine rings spaced  $1/2$  apart and  $d_m H^{1/2}$ .

TABLE I.

RING RADIUS AND NUMBER OF ELEMENTS PER RING FOR A 9-RING CONCENTRIC CIRCULAR ANTENNA ARRAY.

M	1	2	3	4	5	6	7	8	9
$r_m(\lambda)$	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
$N_m$	6	12	18	25	31	37	43	50	56

TABLE II.

EXCITATION AMPLITUDE DISTRIBUTION ( $I_{mn}$ ) USING IPSO

Concentric Ring No.	No. of elements in the corresponding ring	Element Excitations corresponding to the Concentric Circle
central element	1	1
1	6	101001
2	12	001000110011
3	18	011100010010001111
4	25	01101100111100110101 11101
5	31	01001101010000100000 0 0110001100
6	37	10110010110000001011 1 0000011101110111
7	43	10000000101001001101 00 00110111010110110 1010
8	50	11110010010010011011 11 10010100011101110 10000 010011
9	56	01010110101011011001 00010010101100000010 1011 101000010100

TABLE III. THINNED AND FULLY POPULATED ARRAY RESULTS

Parameters	IPSO based Synthesized thinned array	Fully populated array
Side Lobe Level (SLL)	-19.5 dB	-17.4 dB
Half-power beamwidth (HPBW, in degree)	6.2	6.2
Number of elements turned off	147	0
Number of elements turned on	132	279

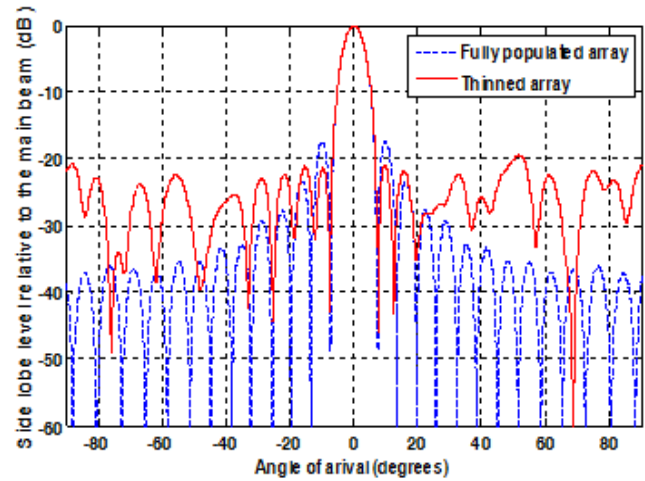


Figure 3. Array pattern found by IPSO for 9 rings CCAA set with central element feeding.

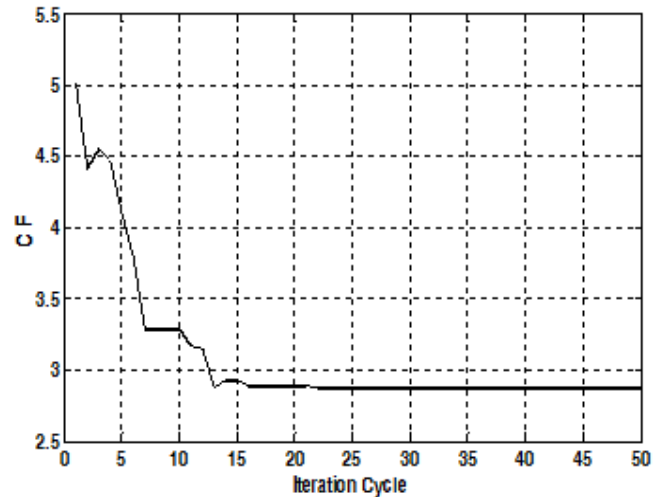


Figure 4. Convergence profile for IPSO in case thinned CCAA

## V. CONCLUSIONS

This paper illustrates a PSO based technique for thinning of large Concentric Circular Antenna Array of isotropic elements. The ultimate objective of the technique is to design an array with a reduction of around 53% of the total elements used in case of a fully populated array with a simultaneous reduction in side lobe level to around 20 dB. The IPSO algorithm can efficiently handle the thinning of a large multiple concentric circular ring arrays of uniformly excited isotropic antennas. The Half Power Beamwidth (HPBW) of the synthesized array pattern with fixed inter-element spacing is remain same to that of a fully populated array of same shape and size, moreover has a better side lobe level.

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